

Walls of massive Kähler sigma models on $SO(2N)/U(N)$ and $Sp(N)/U(N)$

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Abstract

We study the Bogomol'nyi-Prasad-Sommerfield wall solutions in massive Kähler non-linear sigma models on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ in three-dimensional spacetime. We show that $SO(2N)/U(N)$ and $Sp(N)/U(N)$ models have 2^{N-1} and 2^N discrete vacua, respectively. We explicitly construct the exact BPS multiwall solutions for $N \leq 3$.

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1 Introduction

Topological solitons in supersymmetric (SUSY) theories preserve a fraction of the original SUSY [1] as they saturate the Bogomol'nyi-Prasad-Sommerfield (BPS) bound on the energy [2]. Domain walls are one of the simplest BPS objects, which conserve half of SUSY [3, 4, 5].

A systematic method to construct domain wall solutions in SUSY $U(N_C)$ gauge theories coupled to N_F ($N_F > N_C$) massive hypermultiplets in the fundamental representation in the presence of the Fayet-Iliopoulos has been proposed [6, 7]. The mass term forms nontrivial scalar potential, yielding $N_F C_{N_C}$ discrete vacua. Exact domain wall solutions interpolating these vacua have been obtained. This construction method is called the moduli matrix approach. Subsequently this approach has been applied to obtain other types of solitonic solutions such as monopole-vortex-wall systems [8], domain wall webs [9], non-Abelian vortices [10], instanton-vortex systems [11] and Skyrmons [12]. For a comprehensive review, see [13]. The model considered in [6, 7] has an interesting limit: Taking infinite gauge coupling limit, the kinetic terms of gauge fields and their superpartners are vanishing and they turn out to be just a Lagrange multiplier, yielding constraints to hypermultiplets. The action with such a limit gives the quotient action of massive hyper-Kähler (HK) nonlinear sigma model (NLSM) on cotangent bundle over Grassmannian, $T^*G_{N_F, N_C}$. Vacuum structure of this NLSM was originally addressed in [14], giving the same number of vacua before taking infinite gauge coupling limit while domain wall solution was obtained only in simple case, for instance, $T^*G_{2,1} \simeq T^*\mathbf{CP}^1$ [5, 15, 16, 17, 18] until the moduli matrix approach was proposed.

The Grassmann manifold is one of the compact Hermitian symmetric spaces (HSS), which we denote \mathcal{M} . It consists of the four classical types, the Grassmann manifold $G_{N+M, M}$, complex quadric surface $Q^N = SO(N+2)/[SO(N) \times U(1)]$, $SO(2N)/U(N)$, $Sp(N)/U(N)$ and two exceptional types, $E_6/[SO(10) \times U(1)]$ and $E_7/[E_6 \times U(1)]$. It is interesting to investigate vacuum structure and domain wall solutions in HK NLSM on cotangent bundle over \mathcal{M} other than $G_{N+M, M}$ since the other HSS are expected to give rich vacuum structure as well as abundant wall solutions similar to HK NLSM on $T^*G_{N+M, M}$. The actions of HK NLSM on cotangent bundle over classical HSS [19, 20, 21]³ and on $T^*E_6/[SO(10) \times U(1)]$ [23] have been obtained in

³A massless HK NLSM on the tangent bundle over the complex quadric surface being one of the classical

projective superspace [24, 25], but they are not the quotient actions. They are written in terms of physical degrees of freedom without Lagrange multiplier and therefore they are not gauge theories with infinite gauge coupling limit. The moduli matrix approach can be easily applied to a quotient action, but it is difficult to construct a quotient action on cotangent bundle over \mathcal{M} except Grassmann manifold.

In [6, 7], it has been also shown that when considering wall solutions in massive HK NLSM on $T^*G_{N_F, N_C}$ the cotangent part is trivial. Only field coordinates parameterizing base manifold have nontrivial configuration. It means that we can simply drop the cotangent bundle part in massive HK NLSM, reducing the massive HK NLSM to a simpler model, massive Kähler NLSM.

In this paper, we investigate discrete vacua and domain wall solutions interpolating them in massive Kähler NLSM on the HSS, $SO(2N)/U(N)$ and $Sp(N)/U(N)$. These massive NLSMs are obtained from the massless Kähler NLSM on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ in four-dimensional spacetime which are described as a quotient action [26] by dimensional reduction. The actions possess a nontrivial scalar potential which includes mass terms characterized by the common Cartan matrices of $SO(2)$ and $Sp(N)$, leading to many discrete vacua. We show that the $SO(2N)/U(N)$ model possesses 2^{N-1} number of discrete vacua while 2^N number of discrete vacua exists in the $Sp(N)/U(N)$ model. We derive wall solutions interpolating those vacua and also discuss their properties.

Organization of this paper is as follows. In Section 2, we present the massive NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$. We also discuss the discrete vacua induced by the mass term. In Section 3, we derive half BPS equations. In Section 4 and 5, the exact wall solutions are obtained in two models based on the moduli matrix approach. Section 6 is devoted to conclusion and discussion. In Appendix A, we explain the vacuum structure of massive NLSMs on $SO(6)/U(3)$ and $Sp(3)/U(3)$. In Appendix B, we show domain wall solutions in a massive Kähler NLSM on CP^3 .

HSSs has been worked out in [22].

2 Massive Kähler NLSM on $SO(2N)/U(N)$ and $Sp(N)/U(N)$

In this section, we construct massive Kähler NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ in three-dimensional spacetime by dimensional reduction from the massless model in four-dimensional spacetime. We follow the notation of [27].

SUSY gauge theory inducing a quotient action of massless NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ are formulated in terms of $\mathcal{N} = 1$ superfields [26]. These actions are obtained by imposing quadric constraints on Kähler NLSM on the Grassmann manifold $G_{2N,N} = \frac{SU(2N)}{SU(N) \times U(N)}$. $G_{2N,N}$ can be constructed by an $N \times 2N$ matrix chiral superfield $\phi_a^i(x, \theta, \bar{\theta})$ ($a = 1, \dots, N$, $i = 1, \dots, 2N$) and an $N \times N$ matrix vector superfield $V_a^b(x, \theta, \bar{\theta})$ in the adjoint representation of $U(N)$. We introduce an $N \times N$ matrix chiral superfield $\phi_0^{ab}(x, \theta, \bar{\theta})$ as an auxiliary field to impose the F -term constraint. The Lagrangian of massless NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ is

$$\mathcal{L} = \int d^4\theta (\phi_a^i \bar{\phi}_i^b (e^V)_b^a - r^2 V_a^a) + \left(\int d^2\theta \phi_0^{ab} (\phi_b^i J_{ij} \phi_a^{Tj}) + \text{c.c.} \right), \quad (2.1)$$

where r^2 is the Fayet-Iliopoulos parameter and the matrix J is defined by

$$J = \mathbf{1} \otimes \begin{pmatrix} 0 & 1 \\ \epsilon & 0 \end{pmatrix}, \quad \epsilon = \begin{cases} +1 & SO(2N)/U(N) \\ -1 & Sp(N)/U(N). \end{cases} \quad (2.2)$$

All the repeated indices are implicitly summed over. The auxiliary chiral superfield $\phi_0^{ab}(x, \theta, \bar{\theta})$ is in a symmetric (anti-symmetric) rank 2 tensor representation of $SU(N)$

$$\phi_0^T = \epsilon \phi_0, \quad (2.3)$$

with $U(1) (\in U(N))$ charge -2 . The D -term in (2.1) gives a quotient action of Kähler NLSM on $G_{2N,N}$ while the F -term gives constraints, realizing submanifolds $SO(2N)/U(N)$ and $Sp(N)/U(N)$ as quadrics of $G_{2N,N}$. The latter is obtained by the equation of motion of ϕ_0 as

$$\phi_b^i J_{ij} \phi_a^{Tj} = 0. \quad (2.4)$$

For $\epsilon = +1$ case, the corresponding submanifold satisfying (2.4) has not only $SO(2N)$ symmetry but also parity in flavor indices as J in (2.2) is invariant under $O(2N)$ group. The parity will be removed. We will discuss this in detail in later sections.

Next we derive massive Kähler NLSMs on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ from (2.1). As solitonic objects are the interest, we consider only the bosonic part of (2.1) in the following context. We replace the superfields of the Lagrangian with fields expanded by bosonic components

$$\begin{aligned}\phi_a^i(x, \theta, \bar{\theta}) &= \phi_a^i(x) + \theta^2 F_a^i, \\ \phi_0^{ab}(x, \theta, \bar{\theta}) &= \phi_0^{ab}(x) + \theta^2 F_0^{ab}, \\ V_a^b(x, \theta, \bar{\theta}) &= 2\theta\sigma^\mu\bar{\theta}v_\mu + \frac{1}{2}\theta^2\bar{\theta}^2 D,\end{aligned}\tag{2.5}$$

and then we obtain

$$\begin{aligned}\mathcal{L}_{\text{bos } 4D} &= -|D_\mu\phi_a^i|^2 + |F_a^i|^2 + \frac{1}{2}(D_a^b\phi_b^i\bar{\phi}_i^a - D_a^a) \\ &\quad + \left((F_0)^{ab}\phi_b^i J_{ij}\phi_a^{Tj} + (\phi_0)^{ab}F_b^i J_{ij}\phi_a^{Tj} + (\phi_0)^{ab}\phi_b^i J_{ij}F_a^{Tj} + \text{c.c.}\right),\end{aligned}\tag{2.6}$$

where the Greek letter μ denotes a four-dimensional spacetime index. We introduce the mass term by dimensional reduction along the x^3 -direction as follows

$$\frac{\partial\phi_a^i}{\partial x^3} = i\phi_a^j M_j^i, \quad \frac{\partial\bar{\phi}_i^a}{\partial x^3} = -i\bar{M}_i^j \bar{\phi}_j^a,\tag{2.7}$$

where

$$M_j^i = \text{diag}(m_1, m_2, \dots, m_N) \otimes \sigma_3.\tag{2.8}$$

M_i^j is the Cartan matrix of $SO(2N)$ and $Sp(N)$. The components m_i ($i = 1, \dots, N$) are real and positive parameters with a condition $m_i > m_{i+1}$. The mass term breaks the global symmetries to $SO(2)^N$ and $Sp(1)^N$ for each model. We substitute (2.7) into (2.6) to obtain the Lagrangian for massive $SO(2N)/U(N)$ and $Sp(N)/U(N)$ in three dimensions

$$\begin{aligned}\mathcal{L}_{\text{bos } 3D} &= -|D_m\phi_a^i|^2 - |i\phi_a^j M_j^i - i\Sigma_a^b\phi_b^i|^2 + |F_a^i|^2 + \frac{1}{2}(D_a^b\phi_b^i\bar{\phi}_i^a - D_a^a) \\ &\quad + \left((F_0)^{ab}\phi_b^i J_{ij}\phi_a^{Tj} + (\phi_0)^{ab}F_b^i J_{ij}\phi_a^{Tj} + (\phi_0)^{ab}\phi_b^i J_{ij}F_a^{Tj} + \text{c.c.}\right),\end{aligned}\tag{2.9}$$

where $\Sigma = v_3$. A Roman letter index m refers to the first three components of the four-dimensional index μ . The constraints of the Lagrangian are

$$\phi_a^i\bar{\phi}_i^b - \delta_a^b = 0,\tag{2.10}$$

$$\phi_a^i J_{ij}\phi_b^{Tj} = 0.\tag{2.11}$$

Eliminating the auxiliary fields $F_a{}^i$ by its equation of motion

$$F_a{}^i = -2(\bar{\phi}_0)_{ab}\phi^{*b}{}_j J^{ji}, \quad (2.12)$$

we obtain the following scalar potential

$$V = |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b \phi_b{}^i|^2 + 4|(\phi_0)^{ab}\phi_b{}^i|^2. \quad (2.13)$$

The vacuum condition is readily read off as

$$\phi_a{}^j M_j{}^i - i\Sigma_a{}^b \phi_b{}^i = 0, \quad (2.14)$$

$$(\phi_0)^{ab}\phi_b{}^i = 0, \quad (2.15)$$

with the constraints (2.10) and (2.11). The condition (2.15) gives $(\phi_0)^{ab} = 0$ or $\phi_a{}^i = 0$, but the latter solution is inconsistent with (2.10). Taking account of the former, the vacuum condition is reduced to

$$(m_k - \Sigma_a)\phi_a{}^{2k-1} = 0, \quad (2.16)$$

$$(m_k + \Sigma_a)\phi_a{}^{2k} = 0, \quad (2.17)$$

where we have used (2.8). We have diagonalized Σ by using $U(N)$ gauge symmetry as

$$\Sigma = \text{diag}(\Sigma_1, \Sigma_2, \dots, \Sigma_N). \quad (2.18)$$

The indices $k, a = \{1, \dots, N\}$ are not summed over.

We solve the equations (2.16) and (2.17) with the constraints (2.10) and (2.11). Above equations yield $\Sigma_a = \pm m_k$ for some combinations of a and k . First let us consider a solution $\Sigma_{a_1} = m_{k_1}$ with $a_1 = k_1$ for some $a_1(k_1)$. It leads to $\phi_{a_1}^{2k_1-1} \neq 0$ and also $\phi_{a_1}^{2k_1} = 0$ from (2.17). Similarly, considering a solution $\Sigma_{a_1} = -m_{k_1}$ with $a_1 = k_1$ for some $a_1(k_1)$, we have $\phi_{a_1}^{2k_1-1} = 0$ and $\phi_{a_1}^{2k_1} \neq 0$. For another gauge index $a_2 (\neq a_1)$, possible solutions are $\Sigma_{a_2} = \pm m_{k_2}$ with $k_1 = k_2$ and $\Sigma_{a_2} = \pm m_{k_2}$ with $k_1 \neq k_2$. However, the former is not a solution since it forms vacuum expectation value ϕ not satisfying the constraint (2.10) or (2.11) while the latter forms ϕ satisfying the constraints. Similarly, for a_3 we find that $\Sigma_{a_3} = \pm m_{k_3}$ with $k_1 \neq k_3$ and $k_1 \neq k_2$

forms ϕ satisfying the constraints. Repeating the same discussion for the other gauge indices, we have the following Σ for the vacua

$$(\Sigma_{a_1}, \Sigma_{a_2}, \Sigma_{a_3}, \dots, \Sigma_{a_N}) = (\pm m_{k_1}, \pm m_{k_2}, \pm m_{k_3}, \dots, \pm m_{k_N}), \quad (2.19)$$

where $a_1 = k_1$ with $a_i \neq a_j$ and $k_i \neq k_j$ for $i \neq j$. The $U(N)$ gauge symmetry allows us to take (2.19) to the following form

$$(\Sigma_1, \Sigma_2, \Sigma_3, \dots, \Sigma_N) = (\pm m_1, \pm m_2, \pm m_3, \dots, \pm m_N). \quad (2.20)$$

From this expression, we see that the number of possible solutions is 2^N . Since $\epsilon = +1$ in (2.2) defines $O(2N)$ group the half of the solutions are related by parity to the other half. The number of vacua in $SO(2N)/U(N)$ is therefore 2^{N-1} . The number of vacua in $Sp(N)/U(N)$ is 2^N as shown in (2.20).

We consider $N = 2$ case explicitly. There are four solutions satisfying (2.16) and (2.17) with (2.10) and (2.11):

$$\begin{aligned} \Phi_{\langle 1 \rangle} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 \end{pmatrix}, & (\Sigma_1, \Sigma_2) &= (m_1, m_2), \\ \Phi_{\langle 2 \rangle} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 \end{pmatrix}, & (\Sigma_1, \Sigma_2) &= (m_1, -m_2), \\ \Phi_{\langle 3 \rangle} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha_3 & 0 \end{pmatrix}, & (\Sigma_1, \Sigma_2) &= (-m_1, m_2), \\ \Phi_{\langle 4 \rangle} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \alpha_4 \end{pmatrix}, & (\Sigma_1, \Sigma_2) &= (-m_1, -m_2), \end{aligned} \quad (2.21)$$

where $\langle 1 \rangle, \dots, \langle 4 \rangle$ denote the labels of the vacua. $\alpha_i = 1$ ($i = 1, \dots, 4$) for $SO(4)/U(2)$ whereas $\alpha_i = \pm 1$ for $Sp(2)/U(2)$.

For $\epsilon = +1$, the parity in (2.21) can be identified by $O(4)$ group elements. We define a rotation transformation \mathcal{R} and a parity transformation \mathcal{P} of $O(4)$ group as

$$\mathcal{R} = \begin{pmatrix} P & 0 \\ 0 & P \end{pmatrix}, \quad \mathcal{P} = \begin{pmatrix} I & 0 \\ 0 & P \end{pmatrix}, \quad (2.22)$$

where $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and I is a two-by-two identity matrix. The vacua are then related by

$$\Phi_{\langle 1 \rangle} = \Phi_{\langle 4 \rangle} \mathcal{R}, \quad \Phi_{\langle 2 \rangle} = \Phi_{\langle 3 \rangle} \mathcal{R}, \quad \Phi_{\langle 1 \rangle} = \Phi_{\langle 2 \rangle} \mathcal{P}. \quad (2.23)$$

It shows that (2.9) with $\epsilon = +1$ involves two massive $SO(4)/U(2)$ models related by parity. Two sets of two vacua ($\langle 1 \rangle, \langle 4 \rangle$) and ($\langle 2 \rangle, \langle 3 \rangle$) belong to each massive $SO(4)/U(2)$ model. Therefore there exist two discrete vacua in a single massive $SO(4)/U(2)$ model. $SO(4)/U(2)$ is isomorphic to \mathbf{CP}^1 [28]. A massive Kähler NLSM on \mathbf{CP}^1 has only two discrete vacua [29]. The results are consistent.

For $\epsilon = -1$, (2.11) defines an invariant submanifold under the action of $Sp(2)$, leading to a single massive $Sp(2)/U(2)$ model. We find that there are $2^2 = 4$ discrete vacua in this case.

3 BPS equations

In this section, we derive the BPS equation for wall solutions from the Bogomol'nyi completion of the Hamiltonian. We assume that fields are static and all the fields depend only on the $x_1 \equiv x$ coordinate. We also assume Poincaré invariance on the two-dimensional world volume of walls to set $v_0 = v_2 = 0$. The energy along the x -direction is

$$\begin{aligned} E &= \int dx \left(|D\phi_a^i|^2 + |\phi_a^j M_j^i - \Sigma_a^b \phi_b^i|^2 + 4|(\phi_0)^{ab} \phi_b^i|^2 \right) \\ &= \int dx \left(|D\phi_a^i \mp (\phi_a^j M_j^i - \Sigma_a^b \phi_b^i)|^2 + 4|(\phi_0)^{ab} \phi_b^i|^2 \right) \pm T \\ &\geq \pm T, \end{aligned} \quad (3.1)$$

with the constraints (2.10) and (2.11). The covariant derivative is defined by $(D\phi)_a^i = \partial\phi_a^i - iv_a^b \phi_b^i$. The energy is bounded by tension

$$T = \int dx \partial(\phi_a^i M_i^j \bar{\phi}_j^a). \quad (3.2)$$

The energy is saturated when the (anti-)BPS equations are satisfied

$$(D\phi)_a^i \mp (\phi_a^j M_j^i - \Sigma_a^b \phi_b^i) = 0. \quad (3.3)$$

We choose the upper sign so the BPS equation becomes

$$(D\phi)_a^i - (\phi_a^j M_j^i - \Sigma_a^b \phi_b^i) = 0. \quad (3.4)$$

We introduce complex matrix functions $S_a^b(x)$ and $f_a^i(x)$ defined by

$$\Sigma_a^b - i v_a^b = (S^{-1} \partial S)_a^b, \quad \phi_a^i = (S^{-1})_a^b f_b^i. \quad (3.5)$$

Then the BPS equation (3.4) can be rewritten as

$$\partial f_a^i = f_a^j M_j^i, \quad (3.6)$$

of which the solution is

$$f_a^i = H_{0a}^j (e^{Mx})_j^i, \quad (3.7)$$

where H_0 is a complex constant matrix. As this matrix involves the information of vacua and positions of domain walls, it is called the moduli matrix [6, 7].

The BPS solution to (3.4), obtained by combining (3.5) and (3.7) is

$$\phi_a^i = (S^{-1})_a^b H_{0b}^j (e^{Mx})_j^i. \quad (3.8)$$

From the definitions (3.5), Σ , v and ϕ are invariant under the transformation

$$S_a'^b = V_a^c S_c^b, \quad H_{0a}^i = V_a^c H_{0c}^i, \quad (3.9)$$

where $V \in GL(N, \mathbf{C})$. The V defines an equivalent class of the sets of the matrix functions and moduli matrices (S, H_0) . This is called the world-volume symmetry [6, 7].

Substituting (3.8) into the constraints (2.10) and (2.11), they become

$$H_{0a}^i (e^{2Mx})_i^j H_{0j}^{\dagger b} = (S \bar{S})_a^b \equiv \Omega_a^b, \quad (3.10)$$

$$H_{0a}^i J_{ij} H_b^{\text{T}j} = 0. \quad (3.11)$$

The constraint (3.11) together with the world-volume symmetry (3.9) gives a definition of $SO(2N)/U(N)$ and $Sp(N)/U(N)$. Therefore H_0 parameterizes these manifolds.

In order to analyze the BPS equation, it is useful to consider the gauge invariant quantities [7]. One of the quantities is the identity component of Σ

$$\Sigma_0 = \frac{1}{4} \text{tr}(S^{-1}(\partial\Omega)\Omega^{-1}S) = \frac{1}{4} \frac{\partial \det \Omega}{\det \Omega}, \quad (3.12)$$

where

$$\Sigma_0 = \text{tr}(\Sigma) = \Sigma_1 + \Sigma_2 + \cdots + \Sigma_N. \quad (3.13)$$

The other is the tension (3.2) in terms of Ω

$$T = \int dx \partial(\phi_a^i M_i^j \bar{\phi}_j^a) = \frac{1}{2} \text{tr} \partial(\Omega^{-1} \partial \Omega) = \frac{1}{2} \partial^2 \ln \det \Omega. \quad (3.14)$$

We will use these quantities when we analyze the BPS domain wall solutions.

4 Wall solution in $SO(2N)/U(N)$ model

In this section we construct explicit BPS domain wall solutions for massive $SO(2N)/U(N)$.

4.1 $N = 2$ case

We have shown that there exist two discrete vacua in $SO(2N)/U(N)$ in Section 2. We choose the vacua $\langle 1 \rangle$ and $\langle 4 \rangle$ in (2.21) without loss of generality. We can express these vacua by the moduli matrix H_0 :

$$H_{0\langle 1 \rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad H_{0\langle 4 \rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (4.1)$$

which are related by (3.8). Since there are two discrete vacua, there is only one wall interpolating them. It should be an elementary wall.

Before studying wall solutions in $SO(2N)/U(N)$, we briefly review the property of walls in the Grassmann manifold [7]. In the paper walls are constructed algebraically from elementary walls. By definition, an elementary wall connects two nearest vacua of the same color index

changing the flavor by one unit $i \leftarrow i + 1$. An elementary wall carrying tension $T_{\langle i \leftarrow i+1 \rangle}$ is defined by

$$[cM, a_i] = c(m_i - m_{i+1})a_i = T_{\langle i \leftarrow i+1 \rangle} a_i, \quad (4.2)$$

where c is a constant, M is the mass matrix and a_i is an $N_f \times N_f$ square matrix generating an elementary wall. N_f is the number of the flavors. From the first equality the mass matrix M and the matrix a_i can be interpreted as a Cartan generator and a step operator respectively. The a_i has a nonzero component only in the $(i, i+1)$ -th element, which is equal to a unit. With the use of a_i , an elementary wall is defined by $H_{0\langle A \leftarrow B \rangle} = H_{0\langle A \rangle} e^{a_i(r)}$ where $a_i(r) \equiv e^r a_i (r \in \mathbf{C})$ and $\langle A \rangle$ and $\langle B \rangle$ are the vacua in the flavor i and $i + 1$ respectively in the same color. The $e^{a_i(r)}$ is called the elementary-wall operator [7].

We now turn to the $SO(2N)/U(N)$ model. Elementary walls changing the flavor by one unit for the same color cannot be defined consistently on the $SO(2N)/U(N)$ manifold. The moduli matrices of elementary walls following the definition above do not satisfy the constraint (3.11), which stems from the F -term constraint (2.4). In addition, as it can be seen from the vacua (4.1) of $SO(4)/U(2)$, changing the flavor in the same color by one unit does not lead to the other vacuum.

We shall modify the formalism of [7] slightly for $SO(2N)/U(N)$ case. We introduce an additional element with an opposite sign into step operators a_i . We can construct elementary walls constrained by (3.11). We also choose $U(N)$ gauge appropriately to label the vacua (2.19) keeping Σ in a diagonal form. This can be formulated in terms of the moduli matrices under the world-volume symmetry (3.9).

The vacuum $H_{0\langle 4 \rangle}$ in (4.1) can be transformed as

$$H_{0\langle 4 \rangle} \rightarrow \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \quad (4.3)$$

The elementary wall connecting the vacua $\langle 1 \rangle$ and $\langle 4 \rangle$ is

$$H_{0\langle 1 \leftarrow 4 \rangle} = \begin{pmatrix} 1 & 0 & 0 & -e^r \\ 0 & e^r & 1 & 0 \end{pmatrix}, \quad r \in \mathbf{C}, \quad -\infty < \text{Re}(r) < \infty. \quad (4.4)$$

We substitute (4.4) into (3.10) and obtain

$$S = \begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix}, \quad \begin{aligned} S_1 &= \sqrt{e^{2m_1x} + e^{-2m_2x+2\text{Re}(r)}}, \\ S_2 &= \sqrt{e^{-2m_1x+2\text{Re}(r)} + e^{2m_2x}}. \end{aligned} \quad (4.5)$$

From (3.8) we have the following solution

$$\phi = \begin{pmatrix} S_1^{-1}e^{m_1x} & 0 & 0 & -S_1^{-1}e^{-m_2x+r} \\ 0 & S_2^{-1}e^{-m_1x+r} & S_2^{-1}e^{m_2x} & 0 \end{pmatrix}. \quad (4.6)$$

This has expected boundaries at $x \rightarrow \pm\infty$. The ϕ in those limits are

$$\begin{aligned} \phi(x \rightarrow +\infty) &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \\ \phi(x \rightarrow -\infty) &= \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \end{aligned} \quad (4.7)$$

The limit $\phi(x \rightarrow -\infty)$ in (4.7) is related to $\Phi_{\langle 4 \rangle}$ in (2.21) by $U(N)$ gauge transformation. It shows that the wall (4.4) connects the vacua $\langle 1 \rangle$ and $\langle 4 \rangle$. This result is expected, considering that $SO(4)/U(2)$ is isomorphic to \mathbf{CP}^1 . A massive NLSM on \mathbf{CP}^1 has two discrete vacua and there exists only one wall, which should be an elementary wall. The results of $SO(4)/U(2)$ and \mathbf{CP}^1 are consistent.

We use elementary-wall operators to construct walls from the next section.

4.2 $N = 3$ case

As shown in Appendix A, there are two sets of vacua ($\langle 1 \rangle, \langle 4 \rangle, \langle 6 \rangle, \langle 7 \rangle$) and ($\langle 2 \rangle, \langle 3 \rangle, \langle 5 \rangle, \langle 8 \rangle$) which are related by parity. We choose the former set of the vacua without loss of generality. We consider the world-volume symmetry to obtain appropriate boundaries of walls as it is done in Section 4.1. The moduli matrices for the vacua are

$$H_{0\langle 1 \rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad H_{0\langle 4 \rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix},$$

$$H_{0\langle 6 \rangle} = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad H_{0\langle 7 \rangle} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (4.8)$$

There are three matrices which generate elementary walls

$$a_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (4.9)$$

$$a_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

We define $a_i(r) \equiv e^r a_i$ to obtain the moduli matrices for elementary walls

$$H_{0\langle 1 \leftarrow 4 \rangle} = H_{0\langle 1 \rangle} e^{a_1(r_1)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -e^{r_1} \\ 0 & 0 & 0 & e^{r_1} & 1 & 0 \end{pmatrix},$$

$$H_{0\langle 4 \leftarrow 6 \rangle} = H_{0\langle 4 \rangle} e^{a_2(r_1)} = \begin{pmatrix} 1 & 0 & -e^{r_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & e^{r_1} & 0 & 1 & 0 & 0 \end{pmatrix},$$

$$H_{0\langle 6 \leftarrow 7 \rangle} = H_{0\langle 6 \rangle} e^{a_3(r_1)} = \begin{pmatrix} 0 & 0 & -1 & 0 & e^{r_1} & 0 \\ 0 & 0 & 0 & -e^{r_1} & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (4.10)$$

where the parameters r_i ($i = 1, 2, \dots$) are complex numbers ranging $-\infty < \text{Re}(r_i) < \infty$. We follow the convention that the operators act on the moduli matrices from the right.

Non-vanishing commutation relations of matrices in (4.9) define compressed single walls. A compressed wall generated by compressing n elementary walls is a compressed wall of level $n - 1$ [7]. There are two compressed walls of level one generated by

$$E_1 = [a_1, a_2] \neq 0, \quad E_2 = [a_2, a_3] \neq 0. \quad (4.11)$$

The matrix E_1 generates a wall, which interpolates the vacua $\langle 1 \rangle$ and $\langle 6 \rangle$ while the matrix E_2 generates a wall, which interpolates the vacua $\langle 4 \rangle$ and $\langle 7 \rangle$:

$$H_{0\langle 1 \leftarrow 6 \rangle} = H_{0\langle 1 \rangle} e^{E_1(r_1)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -e^{r_1} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & e^{r_1} & 0 & 0 & 1 & 0 \end{pmatrix}, \quad (4.12)$$

$$H_{0\langle 4 \leftarrow 7 \rangle} = H_{0\langle 4 \rangle} e^{E_2(r_1)} = \begin{pmatrix} 1 & 0 & 0 & 0 & e^{r_1} & 0 \\ 0 & e^{r_1} & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad (4.13)$$

where $E_i(r) \equiv e^r E_i$. A compressed wall of level one and an elementary wall can be compressed to be a compressed wall of level two. The corresponding wall is generated by the operator E_3

$$E_3 = [a_1, E_2] = [E_1, a_3] \neq 0. \quad (4.14)$$

The first commutator describes compression of the elementary wall $H_{0\langle 1 \leftarrow 4 \rangle}$ and the compressed wall of level one $H_{0\langle 4 \leftarrow 7 \rangle}$. The second commutator describes compression of the compressed wall of level one $H_{0\langle 1 \leftarrow 6 \rangle}$ and the elementary wall $H_{0\langle 6 \leftarrow 7 \rangle}$. As it can also be seen from Figure 1(a), the both of them leads to the same wall interpolating the vacua $\langle 1 \rangle$ and $\langle 7 \rangle$

$$H_{0\langle 1 \leftarrow 7 \rangle} = H_{0\langle 1 \rangle} e^{E_3(r_1)} = \begin{pmatrix} 1 & 0 & 0 & -e^{r_1} & 0 & 0 \\ 0 & e^{r_1} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \quad (4.15)$$

Configurations of the single walls in Σ -space are drawn in Figure 1(a).

Next we consider multiwall solutions. A double wall is constructed by multiplying an

elementary-wall operator to the moduli matrix of a single wall. There are four double walls:

$$H_{0\langle 1 \leftarrow 4 \leftarrow 6 \rangle} = H_{0\langle 1 \leftarrow 4 \rangle} e^{a_2(r_2)} = \begin{pmatrix} 1 & 0 & -e^{r_2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -e^{r_1} \\ 0 & e^{r_1+r_2} & 0 & e^{r_1} & 1 & 0 \end{pmatrix}, \quad (4.16)$$

$$H_{0\langle 4 \leftarrow 6 \leftarrow 7 \rangle} = H_{0\langle 4 \leftarrow 6 \rangle} e^{a_3(r_2)} = \begin{pmatrix} 1 & 0 & -e^{r_1} & 0 & e^{r_1+r_2} & 0 \\ 0 & 0 & 0 & -e^{r_2} & 0 & -1 \\ 0 & e^{r_1} & 0 & 1 & 0 & 0 \end{pmatrix}, \quad (4.17)$$

$$H_{0\langle 1 \leftarrow 4 \leftarrow 7 \rangle} = H_{0\langle 1 \leftarrow 4 \rangle} e^{E_2(r_2)} = \begin{pmatrix} 1 & 0 & 0 & 0 & e^{r_2} & 0 \\ 0 & e^{r_1+r_2} & 1 & 0 & 0 & -e^{r_1} \\ 0 & 0 & 0 & e^{r_1} & 1 & 0 \end{pmatrix}, \quad (4.18)$$

$$H_{0\langle 1 \leftarrow 6 \leftarrow 7 \rangle} = H_{0\langle 1 \leftarrow 6 \rangle} e^{a_3(r_2)} = \begin{pmatrix} 1 & 0 & 0 & -e^{r_1+r_2} & 0 & -e^{r_1} \\ 0 & 0 & 1 & 0 & -e^{r_2} & 0 \\ 0 & e^{r_1} & 0 & 0 & 1 & 0 \end{pmatrix}. \quad (4.19)$$

For instance, $\langle 1 \leftarrow 4 \leftarrow 6 \rangle$ means configuration interpolating three vacua, $\langle 1 \rangle$ at $x = +\infty$, $\langle 4 \rangle$ at $-\infty < x < \infty$ and $\langle 6 \rangle$ at $x = -\infty$. The double walls $H_{0\langle 1 \leftarrow 4 \leftarrow 6 \rangle}$ and $H_{0\langle 4 \leftarrow 6 \leftarrow 7 \rangle}$ are composed of two elementary walls. The double walls $H_{0\langle 1 \leftarrow 4 \leftarrow 7 \rangle}$ and $H_{0\langle 1 \leftarrow 6 \leftarrow 7 \rangle}$ are composed of one elementary wall and one compressed wall of level one as it is described in Figure 1(b). We observe compression of two elementary walls from the double wall $H_{0\langle 1 \leftarrow 4 \leftarrow 6 \rangle}$ as an example. Under the world-volume symmetry, the moduli matrix transforms

$$\begin{aligned} H_{0\langle 1 \leftarrow 4 \leftarrow 6 \rangle} &\rightarrow \begin{pmatrix} 1 & e^{r_2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -e^{r_2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -e^{r_1} \\ 0 & e^{r_1+r_2} & 0 & e^{r_1} & 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -e^{r_1+r_2} \\ 0 & 0 & 1 & 0 & 0 & -e^{r_1} \\ 0 & e^{r_1+r_2} & 0 & e^{r_1} & 1 & 0 \end{pmatrix}. \end{aligned} \quad (4.20)$$

Keeping the parameter $r_1 + r_2$ finite in the limit of $r_1 \rightarrow -\infty$, it leads to a compressed wall of level one, which interpolates the vacua $\langle 1 \rangle$ and $\langle 6 \rangle$. This is illustrated in Figure 2. The upper panels in Figure 2 show the energy density of the multiwall configuration $H_{0\langle 1 \leftarrow 4 \leftarrow 6 \rangle}$. As $r_1 - r_2$

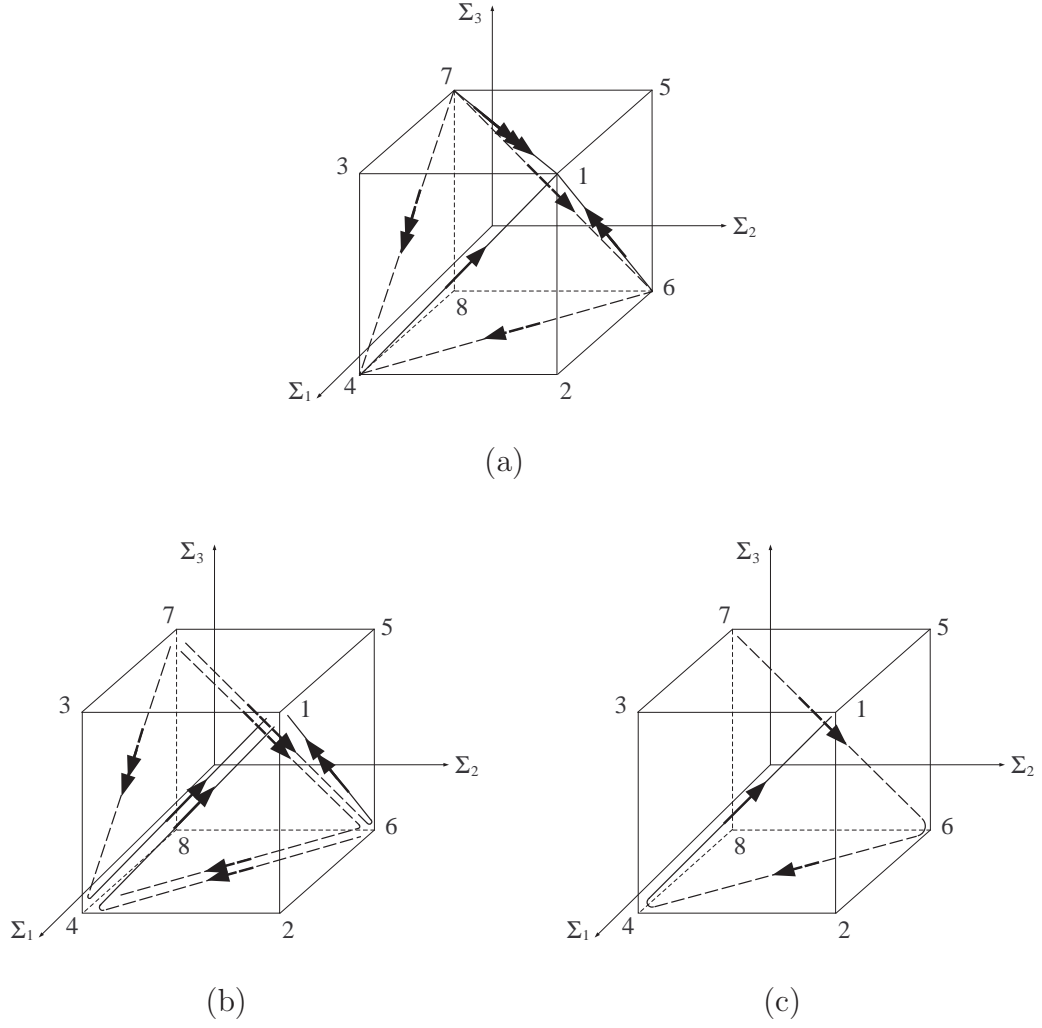


Figure 1: (a)Single walls. Arrows with one arrowhead denote elementary single walls and arrows with two(three) arrowheads denote compressed single walls of level one(two). (b)Double walls. (c)A triple wall.

becomes large, two walls approach and get compressed to a single wall. Such a compression can be also seen in the lower panels in Figure 2. They show the configuration of Σ_0 defined in (3.12) which displays two kinks with the energy density according to r_1 and r_2 .

Finally we consider a triple wall. A triple wall is obtained by multiplying an elementary-wall

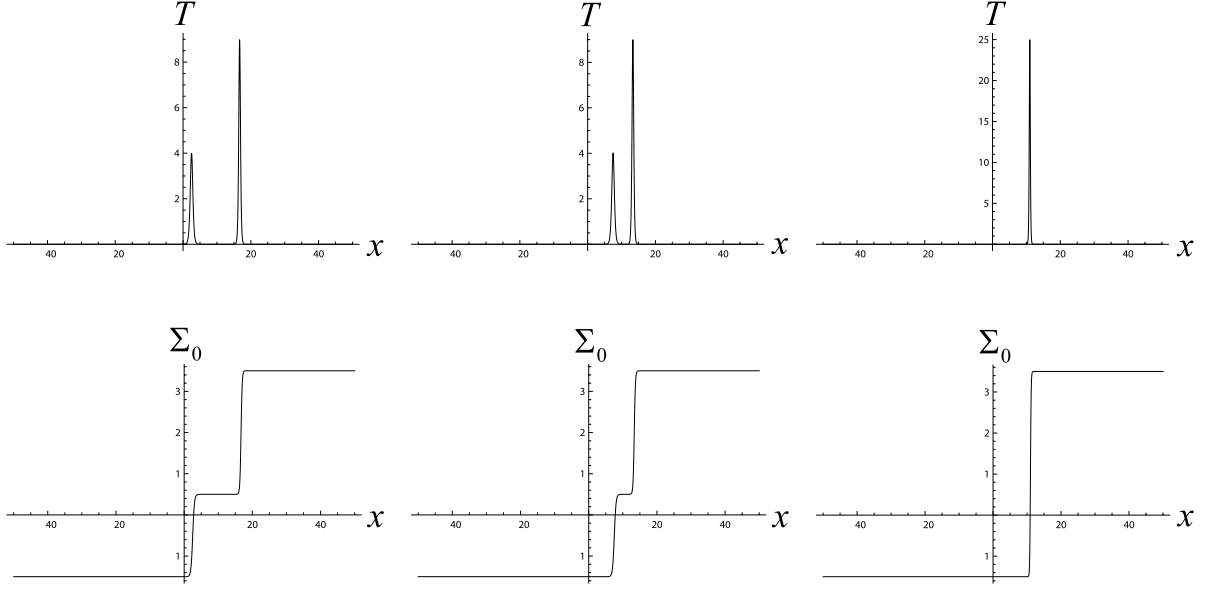


Figure 2: Plots of energy density T and gauge component Σ_0 with masses $m_1 = 4$, $m_2 = 2$, $m_3 = 1$. Two elementary walls forming a double wall become a compressed wall of level two as r_1 and r_2 vary. $(r_1, r_2) = (50, 5)$ in the left, $(r_1, r_2) = (40, 15)$ in the middle and $(r_1, r_2) = (-10, 65)$ in the right.

operator to a moduli matrix for a double wall. There is only one triple wall

$$H_{0\langle 1 \leftarrow 4 \leftarrow 6 \leftarrow 7 \rangle} = H_{0\langle 1 \leftarrow 4 \leftarrow 6 \rangle} e^{a_3(r_3)} = \begin{pmatrix} 1 & 0 & -e^{r_2} & 0 & e^{r_2+r_3} & 0 \\ 0 & 0 & 1 & -e^{r_1+r_3} & -e^{r_3} & -e^{r_1} \\ 0 & e^{r_1+r_2} & 0 & e^{r_1} & 1 & 0 \end{pmatrix}, \quad (4.21)$$

which is composed of three elementary walls as described in Figure 1(c). This is the multiwall composed of the maximal number of single walls in $SO(6)/U(3)$. In $SO(6)/U(3)$, we have found that there are six single walls, consisting of three elementary walls, two compressed wall of level one and two compressed wall of level two, four double walls and one triple wall. We compare this result with \mathbf{CP}^3 , which is isomorphic to $SO(6)/U(3)$ in Appendix B.

5 Wall solution in $Sp(N)/U(N)$ model

In this section we construct explicit BPS domain wall solutions for massive $Sp(N)/U(N)$.

5.1 $N = 1$ case

There are two vacua in $Sp(1)/U(1)$

$$\begin{aligned}\Phi_{\langle 1 \rangle} &= (1, 0), \quad \Sigma = m, \\ \Phi_{\langle 2 \rangle} &= (0, \alpha), \quad \Sigma = -m, \quad (\alpha = \pm 1),\end{aligned}\tag{5.1}$$

from the vacuum condition (2.16) and (2.17). The corresponding moduli matrices are

$$H_{0\langle 1 \rangle} = (1, 0), \quad H_{0\langle 2 \rangle} = (0, 1).\tag{5.2}$$

We have removed the plus-minus sign of α by the world-volume symmetry (3.9). There is only one single wall therefore which is an elementary wall interpolating the vacua

$$H_{0\langle 1 \leftarrow 2 \rangle} = (1, e^r).\tag{5.3}$$

This result is consistent with \mathbf{CP}^1 , which is isomorphic to $Sp(1)/U(1)$.

5.2 $N = 2$ case

The vacua of $Sp(2)/U(2)$ are (2.21). The moduli matrices for the vacua related by (3.8) are

$$\begin{aligned}H_{0\langle 1 \rangle} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad H_{0\langle 2 \rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \\ H_{0\langle 3 \rangle} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad H_{0\langle 4 \rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\end{aligned}\tag{5.4}$$

The plus-minus sign of α_i for $Sp(2)/U(2)$ in (2.21) has been removed by the world-volume symmetry (3.9). Elementary walls interpolating the four vacua are generated by two operators

$$a_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.\tag{5.5}$$

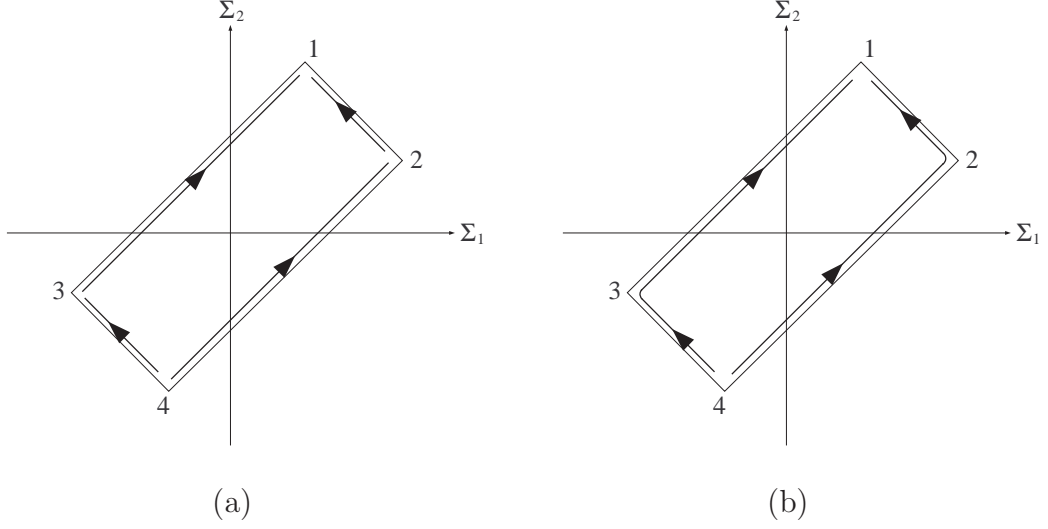


Figure 3: Walls in $Sp(2)/U(2)$. (a) Single walls. (b) Double walls, which are penetrable.

There are twelve elementary single walls

$$\begin{aligned}
H_{0\langle 1 \leftarrow 2 \rangle} &= H_{0\langle 1 \rangle} e^{a_1(r_1)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & e^{r_1} \end{pmatrix}, \\
H_{0\langle 3 \leftarrow 4 \rangle} &= H_{0\langle 3 \rangle} e^{a_1(r_1)} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & e^{r_1} \end{pmatrix}, \\
H_{0\langle 1 \leftarrow 3 \rangle} &= H_{0\langle 1 \rangle} e^{a_2(r_1)} = \begin{pmatrix} 1 & e^{r_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \\
H_{0\langle 2 \leftarrow 4 \rangle} &= H_{0\langle 2 \rangle} e^{a_2(r_1)} = \begin{pmatrix} 1 & e^{r_1} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},
\end{aligned} \tag{5.6}$$

where $a_i(r) \equiv e^r a_i$. The walls are drawn in Figure 3(a). The operators in (5.5) commute:

$$[a_1, a_2] = 0. \tag{5.7}$$

Therefore no compressed wall exists in $Sp(2)/U(2)$.

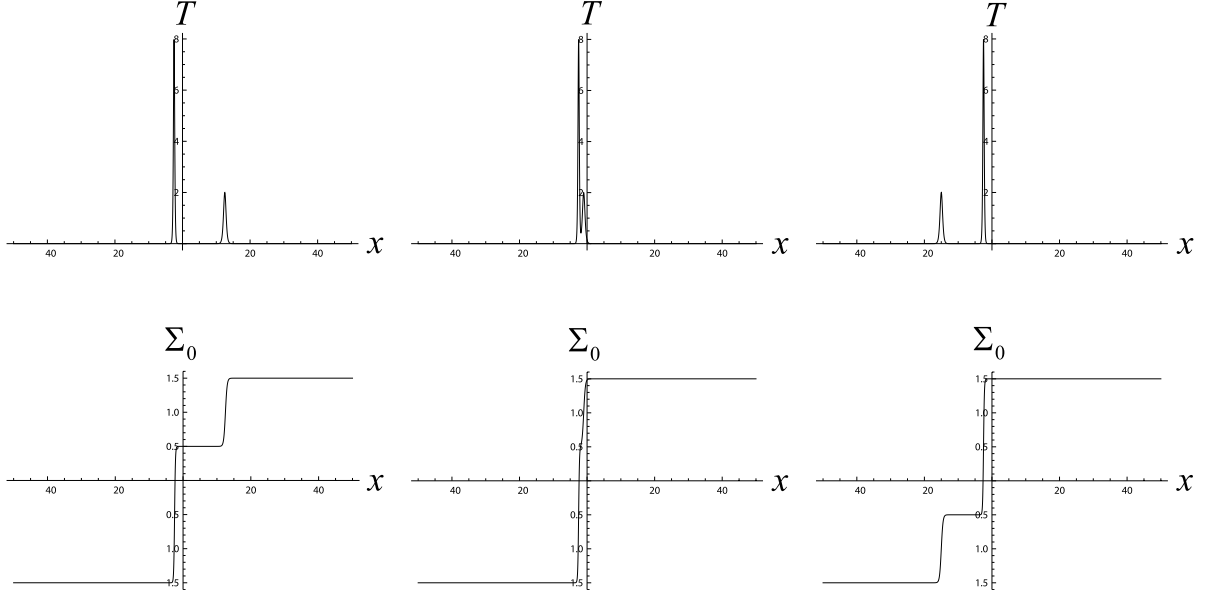


Figure 4: Plots of energy density T and gauge component Σ_0 with $m_1 = 2$, $m_2 = 1$. Two double walls are penetrable. $(r_1, r_2) = (25, -10)$ in the left, $(r_1, r_2) = (-2, -10)$ in the middle and $(r_1, r_2) = (-30, -10)$ in the right.

We construct double walls. There are two double walls

$$\begin{aligned}
 H_{0\langle 1 \leftarrow 2 \leftarrow 4 \rangle} &= H_{0\langle 1 \leftarrow 2 \rangle} e^{a_2(r_2)} = \begin{pmatrix} 1 & e^{r_2} & 0 & 0 \\ 0 & 0 & 1 & e^{r_1} \end{pmatrix}, \\
 H_{0\langle 1 \leftarrow 3 \leftarrow 4 \rangle} &= H_{0\langle 1 \leftarrow 3 \rangle} e^{a_1(r_2)} = \begin{pmatrix} 1 & e^{r_1} & 0 & 0 \\ 0 & 0 & 1 & e^{r_2} \end{pmatrix}.
 \end{aligned} \tag{5.8}$$

The commutation relation (5.7) shows that the double walls in (5.8) form a pair of penetrable walls [7] as shown below

$$\begin{aligned}
 H_{0\langle 1 \leftarrow 2 \leftarrow 4 \rangle} &= H_{0\langle 1 \leftarrow 2 \rangle}(r_1) e^{a_2(r_2)} = H_{0\langle 1 \rangle} e^{a_1(r_1)} e^{a_2(r_2)} \\
 &= H_{0\langle 1 \rangle} e^{a_2(r_2)} e^{a_1(r_1)} = H_{0\langle 1 \leftarrow 3 \rangle}(r_2) e^{a_1(r_1)} \\
 &= H_{0\langle 1 \leftarrow 3 \leftarrow 4 \rangle}.
 \end{aligned} \tag{5.9}$$

The walls go through each other as the parameters r_1 and r_2 vary. This is illustrated in Figure 4.

5.3 $N = 3$ case

There are six vacua in $Sp(3)/U(3)$ as shown in Appendix A. The moduli matrices for the vacua (A.1) can be derived from the relation (3.8):

$$\begin{aligned}
H_{0\langle 1 \rangle} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad H_{0\langle 2 \rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \\
H_{0\langle 3 \rangle} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad H_{0\langle 4 \rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \\
H_{0\langle 5 \rangle} &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad H_{0\langle 6 \rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \\
H_{0\langle 7 \rangle} &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad H_{0\langle 8 \rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5.10}
\end{aligned}$$

The plus-minus signs of α_i and β_i in (A.1) have been removed by the world-volume symmetry (3.9).

There are three operators generating elementary walls

$$a_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$a_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (5.11)$$

Each operator generates four elementary walls

$$\begin{aligned} H_{0\langle 1 \leftarrow 2 \rangle} &= H_{0\langle 1 \rangle} e^{a_1(r_1)}, & H_{0\langle 3 \leftarrow 4 \rangle} &= H_{0\langle 3 \rangle} e^{a_1(r_1)}, \\ H_{0\langle 5 \leftarrow 6 \rangle} &= H_{0\langle 5 \rangle} e^{a_1(r_1)}, & H_{0\langle 7 \leftarrow 8 \rangle} &= H_{0\langle 7 \rangle} e^{a_1(r_1)}, \end{aligned} \quad (5.12)$$

$$\begin{aligned} H_{0\langle 1 \leftarrow 3 \rangle} &= H_{0\langle 1 \rangle} e^{a_2(r_1)}, & H_{0\langle 2 \leftarrow 4 \rangle} &= H_{0\langle 2 \rangle} e^{a_2(r_1)}, \\ H_{0\langle 5 \leftarrow 7 \rangle} &= H_{0\langle 5 \rangle} e^{a_2(r_1)}, & H_{0\langle 6 \leftarrow 8 \rangle} &= H_{0\langle 6 \rangle} e^{a_2(r_1)}, \end{aligned} \quad (5.13)$$

$$\begin{aligned} H_{0\langle 1 \leftarrow 5 \rangle} &= H_{0\langle 1 \rangle} e^{a_3(r_1)}, & H_{0\langle 2 \leftarrow 6 \rangle} &= H_{0\langle 2 \rangle} e^{a_3(r_1)}, \\ H_{0\langle 3 \leftarrow 7 \rangle} &= H_{0\langle 3 \rangle} e^{a_3(r_1)}, & H_{0\langle 4 \leftarrow 8 \rangle} &= H_{0\langle 4 \rangle} e^{a_3(r_1)}, \end{aligned} \quad (5.14)$$

where $a_i(r) \equiv e^r a_i$.

We find that all the operators a_1 , a_2 and a_3 are commutative:

$$[a_1, a_2] = [a_1, a_3] = [a_2, a_3] = 0. \quad (5.15)$$

It shows that there is no compressed wall. All the multiwalls are thereby penetrable in some intermediate regions. Each vanishing commutator in (5.15) describes two pairs of penetrable double walls as follows

$$\begin{aligned} [a_1, a_2] = 0 &\Rightarrow H_{0\langle 1 \leftarrow 2, 3 \leftarrow 4 \rangle}, \quad H_{0\langle 5 \leftarrow 6, 7 \leftarrow 8 \rangle}, \\ [a_1, a_3] = 0 &\Rightarrow H_{0\langle 1 \leftarrow 2, 5 \leftarrow 6 \rangle}, \quad H_{0\langle 3 \leftarrow 4, 7 \leftarrow 8 \rangle}, \\ [a_2, a_3] = 0 &\Rightarrow H_{0\langle 1 \leftarrow 3, 5 \leftarrow 7 \rangle}, \quad H_{0\langle 2 \leftarrow 4, 6 \leftarrow 8 \rangle}. \end{aligned} \quad (5.16)$$

Here, for instance, $\langle 1 \leftarrow 2, 3 \leftarrow 4 \rangle$ means a configuration passing intermediate vacuum 2 or 3 according to a choice of parameters. The triple wall $H_{0\langle 1 \leftarrow 2 \leftarrow 4 \leftarrow 8 \rangle}$ can be constructed by multiplying the other elementary-wall operator $e^{a_3(r)}$ to the double wall $H_{0\langle 1 \leftarrow 2, 3 \leftarrow 4 \rangle} = H_{0\langle 1 \rangle} e^{a_1(r)} e^{a_2(r)}$ as

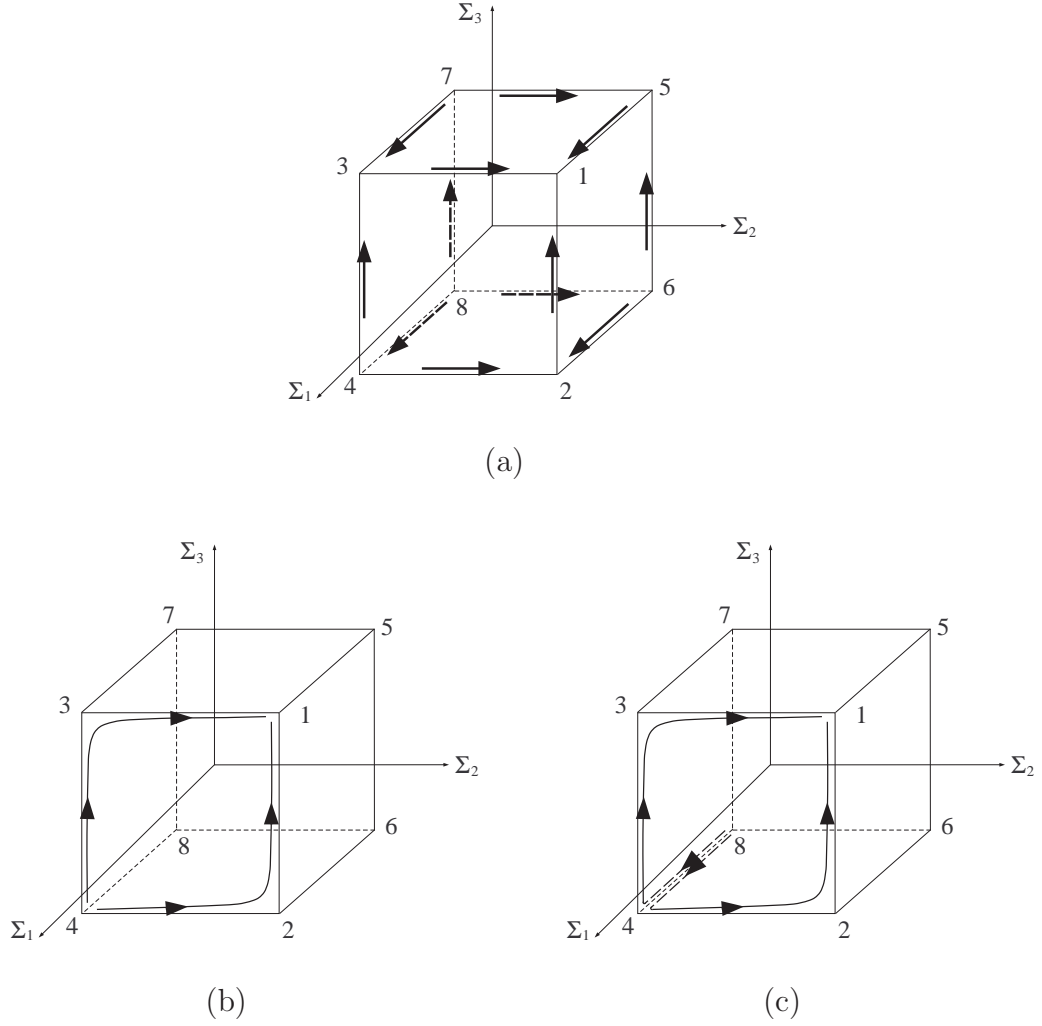


Figure 5: Walls in $Sp(3)/U(3)$. (a) Single walls, which are all elementary walls. (b) A pair of double walls $H_{0\langle 1\leftarrow 2,3\leftarrow 4\rangle}$, which are penetrable. (c) Triple walls $H_{0\langle 1\leftarrow 2,3\leftarrow 4\leftarrow 8\rangle}$.

$$H_{0\langle 1\leftarrow 2\leftarrow 4\leftarrow 8\rangle} = H_{0\langle 1\rangle} e^{a_1(r)} e^{a_2(r)} e^{a_3(r)}. \quad (5.17)$$

Since all the elementary-wall operators are commutative, the triple wall (5.17) describes all the six triple walls $H_{0\langle 1\leftarrow 2\leftarrow 4\leftarrow 8\rangle}$, $H_{0\langle 1\leftarrow 2\leftarrow 6\leftarrow 8\rangle}$, $H_{0\langle 1\leftarrow 3\leftarrow 4\leftarrow 8\rangle}$, $H_{0\langle 1\leftarrow 3\leftarrow 7\leftarrow 8\rangle}$, $H_{0\langle 1\leftarrow 5\leftarrow 6\leftarrow 8\rangle}$ and $H_{0\langle 1\leftarrow 5\leftarrow 7\leftarrow 8\rangle}$. They are penetrable each other in the sense that one or two intermediate vacua interchange. The triple walls are the multiwalls composed of the maximal number of single walls in $Sp(3)/U(3)$. The elementary walls, a pair of double walls $H_{0\langle 1\leftarrow 2,3\leftarrow 4\rangle}$ and triple walls

$H_{0\langle 1\leftarrow 2, 3\leftarrow 4\leftarrow 8 \rangle}$ are drawn in Figure 5.

6 Conclusion

We have studied the vacuum structure and domain walls interpolating the vacua in the massive Kähler NLSM on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ in three-dimensional spacetime in the moduli matrix approach. The mass term has been introduced by dimensional reduction from the massless $\mathcal{N} = 1$ Kähler NLSM in four-dimensional spacetime.

For $SO(2N)/U(N)$ case, we have found that there exist 2^{N-1} discrete vacua. We have studied wall solutions interpolating those vacua explicitly for $N = 2$ and 3 cases. Elementary walls in [7] are defined by positive step operators changing the flavor for the same color by one unit in the Grassmann manifold. In $SO(2N)/U(N)$, however the elementary walls of the definition are not compatible with the F -term constraint, which defines $SO(2N)$ group. We have modified the formalism in the Grassmann manifold slightly. We have introduced an additional unit element in the step operators for elementary walls taking account of the world-volume symmetry of the moduli matrices for vacua which correspond to the boundaries of the elementary walls. For $N = 2$ case, we have shown that there is only one elementary wall, as expected from \mathbf{CP}^1 model, which is isomorphic to $SO(4)/U(2)$. For $N = 3$ case, we have obtained the wall solutions up to triple walls. We have compared the result with \mathbf{CP}^3 , which is isomorphic to $SO(6)/U(3)$. From this, we have shown that the method used for $SO(2N)/U(N)$ does not compromise the algebras of operators generating walls.

For $Sp(N)/U(N)$ case, we have found that there exist 2^N discrete vacua. We have studied wall solutions interpolating those vacua for $N = 1, 2$ and 3 cases. The formalism in [7] has been consistently applied to the models in the presence of the F -term constraint in this case. We have shown that there is a single elementary wall for $N = 1$ case. We have also shown that there exist multiwalls up to double walls and triple walls for $N = 2$ and $N = 3$ cases, respectively.

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Appendix A Vacuum structure of $N = 3$ case

In this Appendix, we derive the vacuum structure for $N = 3$ case. There are eight vacua according to (2.20):

$$\begin{aligned}
\Phi_{\langle 1 \rangle} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_1 & 0 \end{pmatrix}, \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (m_1, m_2, m_3), \\
\Phi_{\langle 2 \rangle} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_2 \end{pmatrix}, \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (m_1, m_2, -m_3), \\
\Phi_{\langle 3 \rangle} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_3 & 0 \end{pmatrix}, \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (m_1, -m_2, m_3), \\
\Phi_{\langle 4 \rangle} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_4 \end{pmatrix}, \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (m_1, -m_2, -m_3), \\
\Phi_{\langle 5 \rangle} &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_5 & 0 \end{pmatrix}, \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (-m_1, m_2, m_3), \\
\Phi_{\langle 6 \rangle} &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_6 \end{pmatrix}, \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (-m_1, m_2, -m_3), \\
\Phi_{\langle 7 \rangle} &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_7 & 0 \end{pmatrix}, \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (-m_1, -m_2, m_3), \\
\Phi_{\langle 8 \rangle} &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_8 \end{pmatrix}, \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (-m_1, -m_2, -m_3). \quad (\text{A.1})
\end{aligned}$$

where $\alpha_i = 1$ and $\beta_i = 1 (i = 1, \dots, 8)$ for $SO(6)/U(3)$ whereas $\alpha_i = \pm 1$ and $\beta_i = \pm 1$ for $Sp(3)/U(3)$.

For $\epsilon = +1$, the half of them are parity-related to the other half as in $N = 2$ case. We define rotation transformations \mathcal{R}_i ($i = 1, 2, 3$)

$$\mathcal{R}_1 = \begin{pmatrix} I & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}, \quad \mathcal{R}_2 = \begin{pmatrix} P & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & P \end{pmatrix}, \quad \mathcal{R}_3 = \begin{pmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & I \end{pmatrix}, \quad (\text{A.2})$$

and a parity transformation of $O(6)$ group,

$$\mathcal{P} = \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & P \end{pmatrix}, \quad (\text{A.3})$$

where I and P are the same as defined by (2.22). The eight vacua (A.1) are related by the rotational transformations as

$$\begin{aligned} \Phi_{\langle 1 \rangle} &= \Phi_{\langle 4 \rangle} \mathcal{R}_1, & \Phi_{\langle 6 \rangle} &= \Phi_{\langle 7 \rangle} \mathcal{R}_1, \\ \Phi_{\langle 1 \rangle} &= \Phi_{\langle 6 \rangle} \mathcal{R}_2, & \Phi_{\langle 4 \rangle} &= \Phi_{\langle 7 \rangle} \mathcal{R}_2, \\ \Phi_{\langle 1 \rangle} &= \Phi_{\langle 7 \rangle} \mathcal{R}_3, & \Phi_{\langle 4 \rangle} &= \Phi_{\langle 6 \rangle} \mathcal{R}_3, \end{aligned} \quad (\text{A.4})$$

while the vacua $\Phi_{\langle 1 \rangle}$ and $\Phi_{\langle 2 \rangle}$ are related by the parity transformation as

$$\Phi_{\langle 1 \rangle} = \Phi_{\langle 2 \rangle} \mathcal{P}. \quad (\text{A.5})$$

It shows that the vacua $\Phi_{\langle 1 \rangle}$, $\Phi_{\langle 4 \rangle}$, $\Phi_{\langle 6 \rangle}$ and $\Phi_{\langle 7 \rangle}$ are in one $SO(6)/U(3)$ manifold and the rest of the vacua are in the other $SO(6)/U(3)$ manifold. Focusing on one of the manifolds, we obtain four discrete vacua.

For $\epsilon = -1$, all the vacua (A.1) are in a single $Sp(3)/U(3)$ model since the constraint (2.11) is an invariant submanifold under the action of $Sp(3)$ group. We therefore find that there exist $2^3 = 8$ discrete vacua in the $Sp(3)/U(3)$ model.

Appendix B CP^3

In this Appendix, we study the vacuum structure and domain walls interpolating them in a massive Kähler NLSM on CP^3 . The action is given by (2.9) with $U(1)$ gauge symmetry and vanishing F -term constraint

$$\mathcal{L}_{\text{bos } 3D} = -|D_m \phi^i|^2 - |i\phi^j M_j^i - i\Sigma \phi^i|^2 + |F^i|^2 + \frac{1}{2}D(\phi^i \bar{\phi}_i - 1). \quad (\text{B.1})$$

The mass matrix M_i^j is given by a linear combination of Cartan matrices for $SU(4)$

$$\frac{1}{\sqrt{2}}\text{diag}(1, -1, 0, 0), \quad \frac{1}{\sqrt{6}}\text{diag}(1, 1, -2, 0), \quad \frac{1}{2\sqrt{3}}\text{diag}(1, 1, 1, -3), \quad (\text{B.2})$$

leading to

$$M = \text{diag}(m'_1, m'_2, m'_3, m'_4), \quad (\text{B.3})$$

where

$$\begin{aligned} m'_1 &= m_1 + m_2 + m_3, \\ m'_2 &= -m_1 + m_2 + m_3, \\ m'_3 &= -2m_2 + m_3, \\ m'_4 &= -3m_3. \end{aligned} \quad (\text{B.4})$$

The mass parameters have been scaled as $\frac{1}{\sqrt{2}}m_1 \rightarrow m_1$, $\frac{-1}{\sqrt{2}}m_2 \rightarrow m_2$, $\frac{-2}{\sqrt{6}}m_3 \rightarrow m_3$ and $\frac{1}{2\sqrt{3}}m_4 \rightarrow m_4$.

Let us see the vacuum structure and domain wall solutions. There are four vacua of which the moduli matrices are

$$\begin{aligned} H_{0\langle 1 \rangle} &= (1, 0, 0, 0), \quad \Sigma = m'_1, \\ H_{0\langle 2 \rangle} &= (0, 1, 0, 0), \quad \Sigma = m'_2, \\ H_{0\langle 3 \rangle} &= (0, 0, 1, 0), \quad \Sigma = m'_3, \\ H_{0\langle 4 \rangle} &= (0, 0, 0, 1), \quad \Sigma = m'_4. \end{aligned} \quad (\text{B.5})$$

We assume that $m'_i > m'_{i+1}$. Operators generating elementary walls are obtained as

$$a'_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad a'_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad a'_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (\text{B.6})$$

The operators (B.6) generate three elementary walls

$$\begin{aligned} H_{0\langle 1 \leftarrow 2 \rangle} &= H_{0\langle 1 \rangle} e^{a'_1(r)} = (1, e^r, 0, 0), \\ H_{0\langle 2 \leftarrow 3 \rangle} &= H_{0\langle 2 \rangle} e^{a'_2(r)} = (0, 1, e^r, 0), \\ H_{0\langle 3 \leftarrow 4 \rangle} &= H_{0\langle 3 \rangle} e^{a'_3(r)} = (0, 0, 1, e^r). \end{aligned} \quad (\text{B.7})$$

Non-vanishing commutators of the matrices in (B.6) are

$$E'_1 = [a'_1, a'_2] \neq 0, \quad E'_2 = [a'_2, a'_3] \neq 0, \quad (\text{B.8})$$

which generate compressed walls of level one

$$H_{0\langle 1 \leftarrow 3 \rangle} = H_{0\langle 1 \rangle} e^{E'_1} = (1, 0, e^r, 0), \quad (\text{B.9})$$

$$H_{0\langle 2 \leftarrow 4 \rangle} = H_{0\langle 2 \rangle} e^{E'_2} = (0, 1, 0, e^r). \quad (\text{B.10})$$

The commutators among operators generating elementary walls and compressed walls of level one are

$$E'_3 = [a'_1, E'_2] = [E'_1, a'_3] \neq 0, \quad (\text{B.11})$$

which generates a compressed wall of level two

$$H_{0\langle 1 \leftarrow 4 \rangle} = H_{0\langle 1 \rangle} e^{E'_3} = (1, 0, 0, e^r). \quad (\text{B.12})$$

We give moduli matrices of multiwalls. Double walls are

$$\begin{aligned} H_{0\langle 1 \leftarrow 2 \leftarrow 3 \rangle} &= H_{0\langle 1 \leftarrow 2 \rangle} e^{a'_2(r_2)} = (1, e^{r_1}, e^{r_1+r_2}, 0), \\ H_{0\langle 2 \leftarrow 3 \leftarrow 4 \rangle} &= H_{0\langle 2 \leftarrow 3 \rangle} e^{a'_3(r_2)} = (0, 1, e^{r_1}, e^{r_1+r_2}), \\ H_{0\langle 1 \leftarrow 2 \leftarrow 4 \rangle} &= H_{0\langle 1 \leftarrow 2 \rangle} e^{E'_2(r_2)} = (1, e^{r_1}, 0, e^{r_1+r_2}), \\ H_{0\langle 1 \leftarrow 3 \leftarrow 4 \rangle} &= H_{0\langle 1 \leftarrow 3 \rangle} e^{a'_3(r_2)} = (1, 0, e^{r_1}, e^{r_1+r_2}). \end{aligned} \quad (\text{B.13})$$

There is one triple wall

$$H_{0\langle 1\leftarrow 2\leftarrow 3\leftarrow 4\rangle} = H_{0\langle 1\leftarrow 2\leftarrow 3\rangle} e^{a'_3(r_3)} = (1, e^{r_1}, e^{r_1+r_2}, e^{r_1+r_2+r_3}). \quad (\text{B.14})$$

We compare the vacuum structure and the algebras of operators of $SO(6)/U(3)$ in Section 4.2 with the result of \mathbf{CP}^3 . The number of vacua and the number of generating operators for elementary walls of $SO(6)/U(3)$ are the same as each of \mathbf{CP}^3 . The algebras for compressed walls (4.11) and (4.14) are also the same as (B.8) and (B.11). The vacuum structure and wall configuration of $SO(6)/U(3)$ coincide with those of \mathbf{CP}^3 under the relabeling of the vacua $\langle 1 \rangle$ to $\langle 1 \rangle$, $\langle 4 \rangle$ to $\langle 2 \rangle$, $\langle 6 \rangle$ to $\langle 3 \rangle$ and $\langle 7 \rangle$ to $\langle 4 \rangle$.

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